Instrument Modeling:
Photon Counting with EMCCD's

Bijan Nemati

Tellus1 Scientific, LLC

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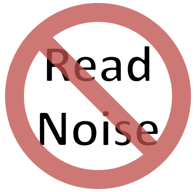
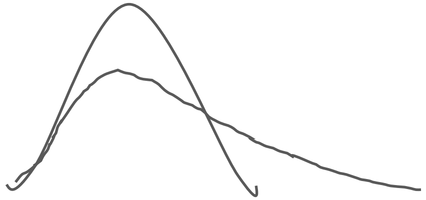
Why EMCCD's and why photon counting?

- Detection of planets requires suppressing the starlight by many orders of magnitude
  - This creates a "dark hole" where the photon rates are very low
  - The planet photon rate is also very low (of order milli-photons/sec/pixel)
  - Under such conditions **detector noise** can become a very important source of error
- Many ultra-low noise detector technologies are being considered for the HWO
  - But the highest TRL architecture by far is the EMCCD – it is being flown on the Roman coronagraph
- Here we discuss how to use the EMCCD under these conditions

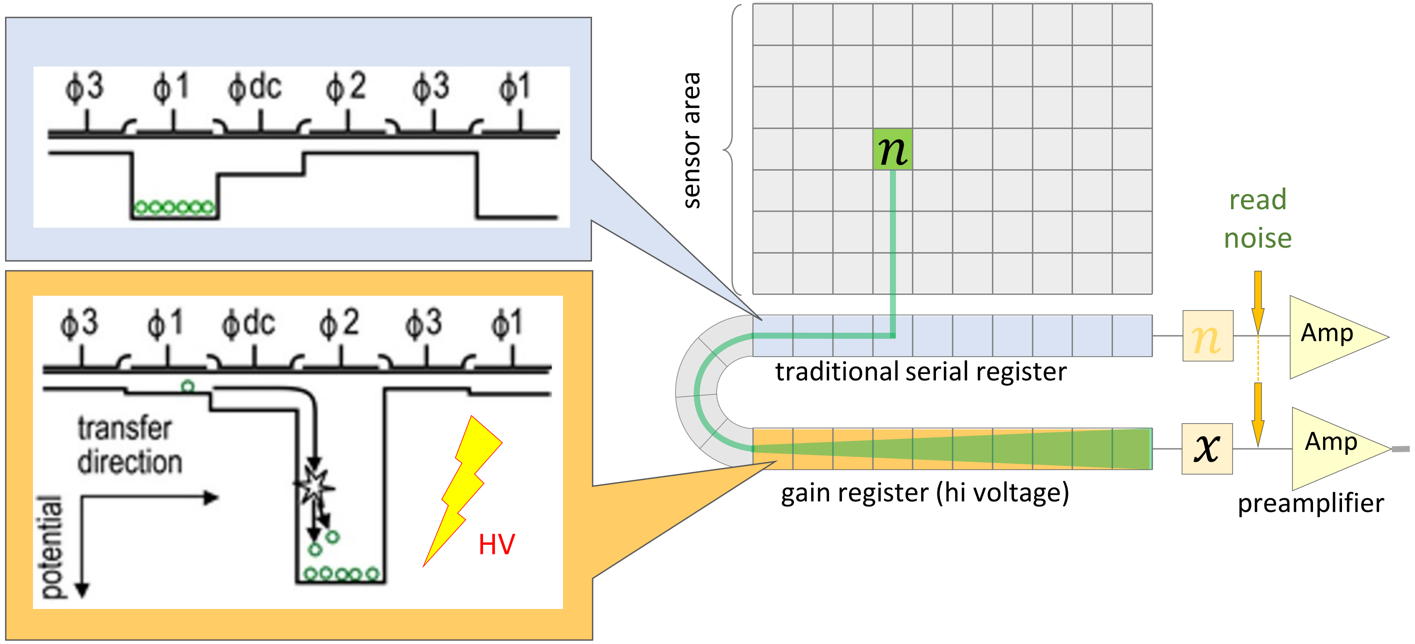
Electron Multiplication (EM) CCD’s



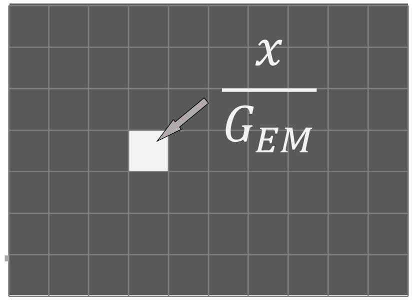
- In an **EMCCD**, pixel charge packets are routed through a multiplication register with a high-voltage phase (10’s of V) where they undergo multiplication
- At each gain stage there is a small (typically < 2%) chance of getting an extra electron (i.e. multiplication)
- Since there are hundreds of multiplication elements, there can be a large gain:



$$G_{EM} = (1+p)^N \quad \text{e.g.} \quad (1+1.5\%)^{600} \simeq 7500$$



We recover the counts by dividing the raw image by the gain G_{EM}



This produces what we could call a “proportional” or “analog” frame

Roman CGI EMCCD Detectors

- CCD311 (based on the CCD201):
  - Removes store shield
  - Implements a single “notch channel” design in the image area
  - Adds an overspill feature to the gain register
  - Implements a new output stage to reduce noise with higher output loads

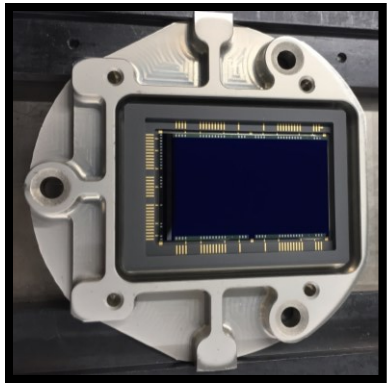
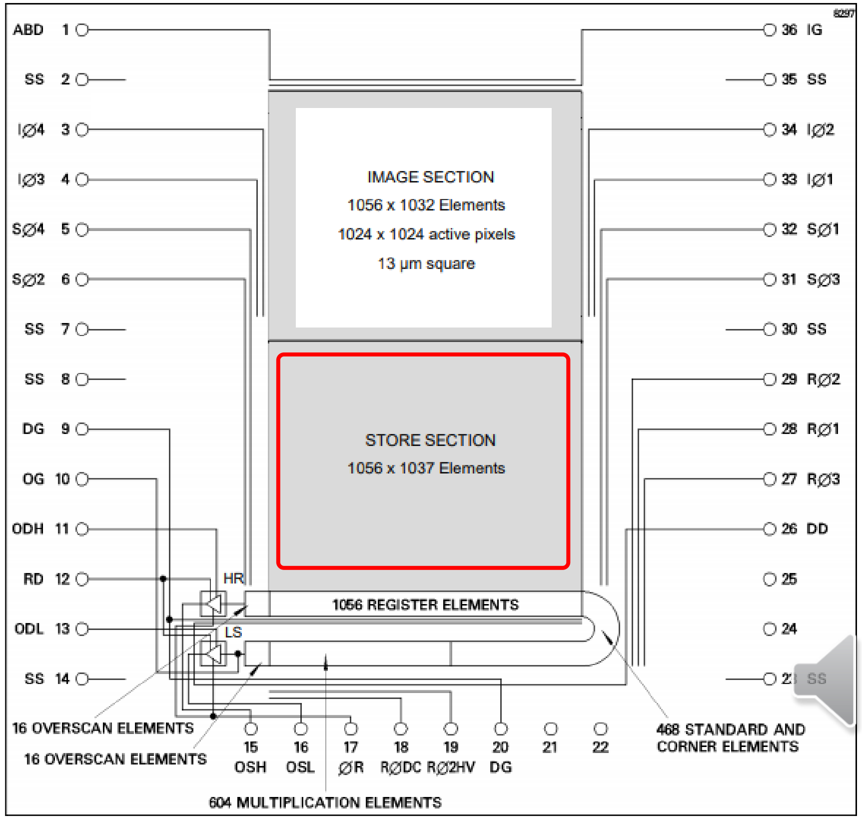


Figure 1: SCHEMATIC CHIP DIAGRAM



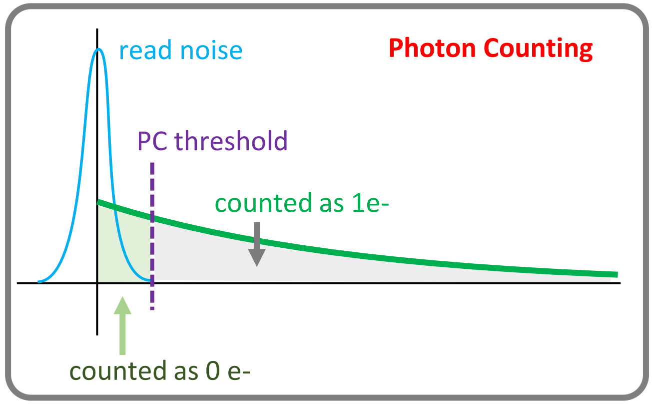
From normal CCD to EMCCD with photon counting

- Normally the detector noise contributions are
  - read noise, dark current, clock-induced charge
- With extremely faint signals and a normal CCD,
  - **read noise** would be **dominant**

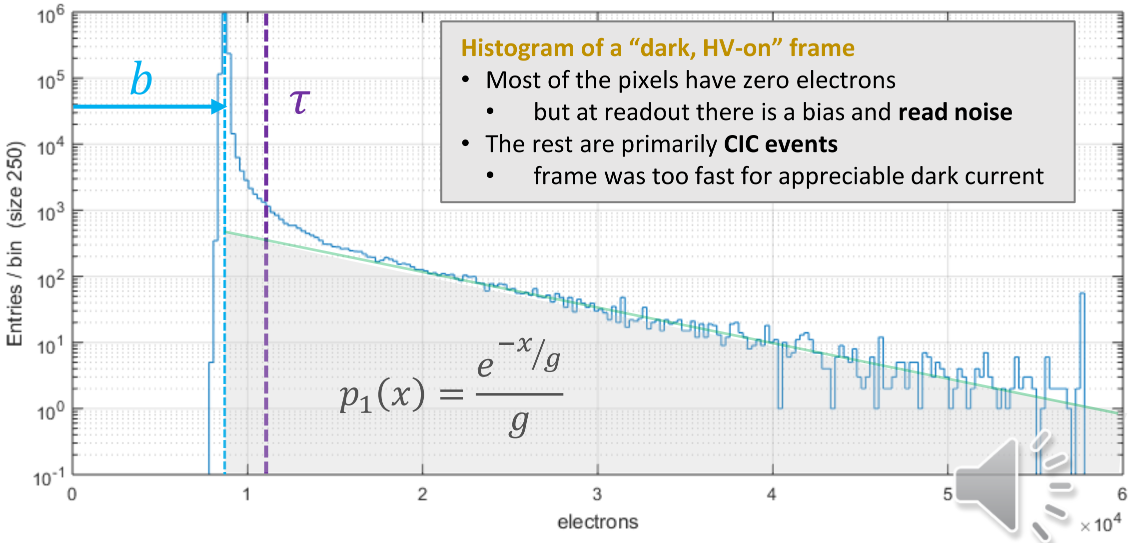
	Normal CCD
Advantages	Well known
Disadvantages	dominant measurement noise is read noise

Photon Counting with EMCCD's

- Increase frame rate until:
  - most pixels have 0 or 1 electron
    - a good rule is to aim for $\sim 0.1e^{-}/fr$



- Set a threshold $\tau = b + k \cdot \sigma_r$
  - b is the bias
  - k is typically ~5-6
  - σ_r is the read noise



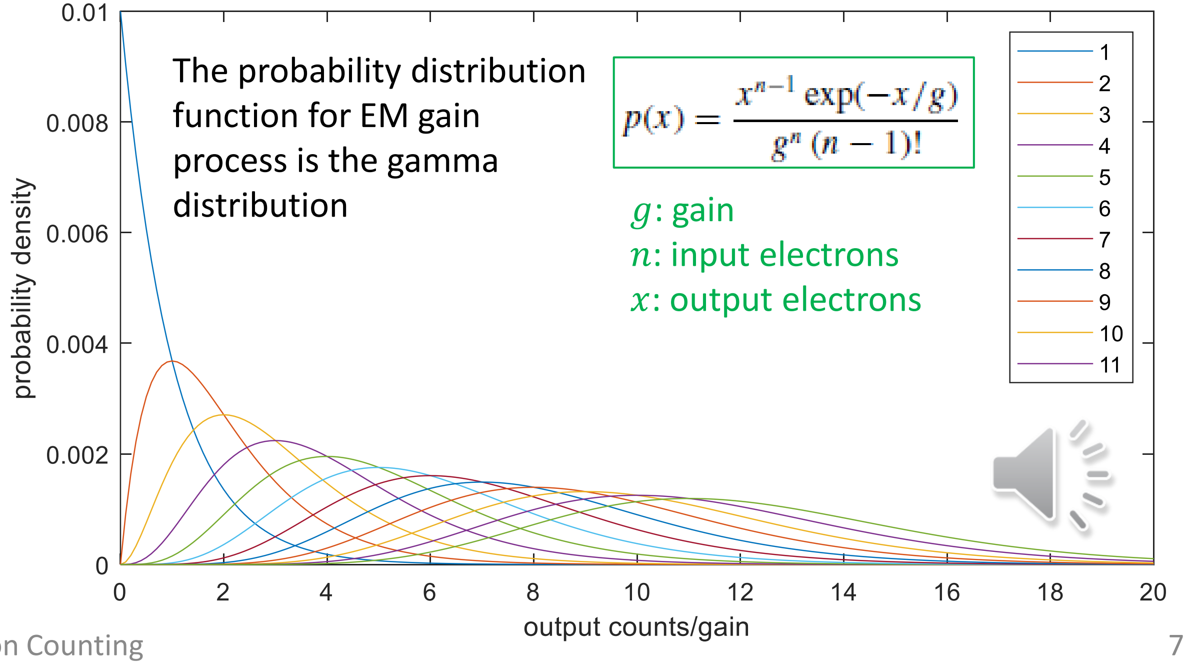
- Every pixel with counts > 1:
  - is deemed to have exactly 1 count
  - else zero counts

Photometric Corrections When Photon Counting – I

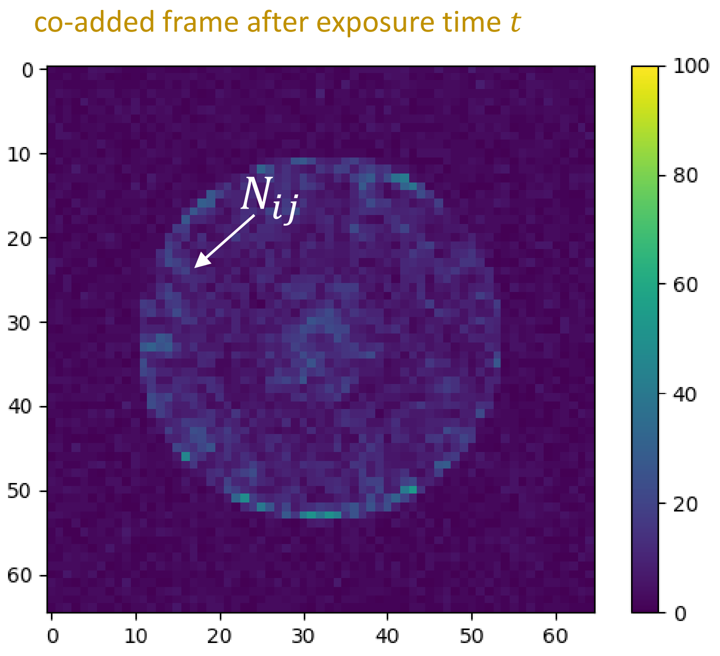
- There are a number of systematic errors that occur when photon counting:
  - RN: 0→1 overcount
  - Thresh: 1→0, 2→0,… undercount
  - Coinc: 2→1, 3→1,… undercount

- In accounting for these, the literature typically accounts only for the dominant term, shown in red.

- Bleed-in from read noise is mitigated by setting a high enough threshold. The level is set by the allowable false positive rate.
- Thresholding loss occurs when we assume zero counts when there actually was 1 (or more) image electrons from any source
- Coincidence loss occurs when in photon counting we take as 1 count a case with more counts



Photon Counting Procedure



1. shorten the single-frame exposure time t_f until most (e.g.~90%) of the pixels have 0 photo-electrons
2. choose a threshold τ for photon counting, such that the SNR is maximized
3. collect n_{fr} bright frames
   - later, also follow the same procedure to get dark frames
4. threshold each frame:
   1. set the count at each pixel to 1 if the analog counts are above τ
   2. otherwise, 0
5. co-add the bright frames to get a single photon counted frame for the full exposure time $t = n_{fr}\, t_f$
6. apply photometric correction starting from the relation:

"photon counting equation" $$N_{ij} = \lambda_{ij}\, n_{fr}\, \epsilon_{th}\, \epsilon_{CL}$$

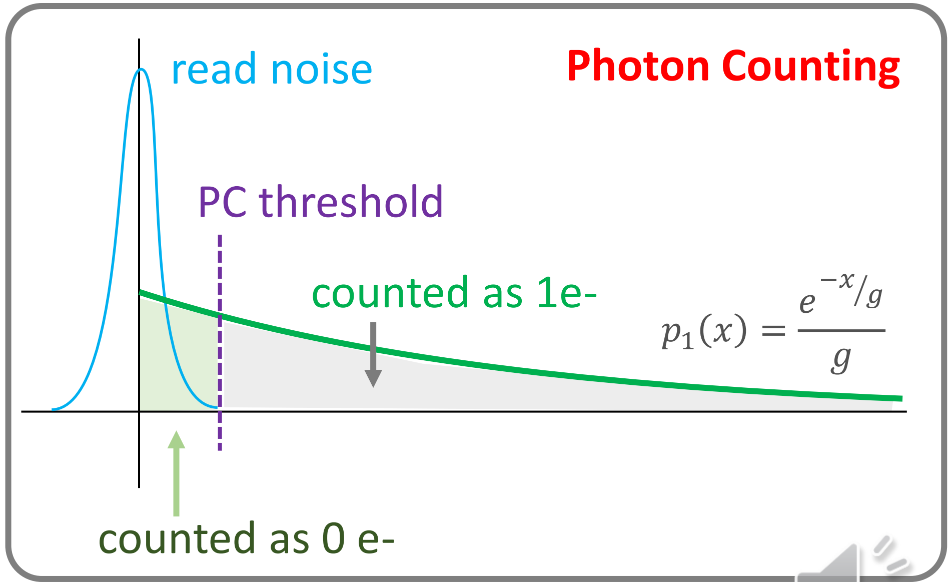
λ is the mean expected rate per frame, for pixel (i, j)
This is what we are after!

Inefficiency factor ϵ_{th}: Thresholding loss

- Applying a threshold means some real events are lost
- Since most events are single photons, the efficiency is very nearly governed by $p_1(x)$

$$\epsilon_{th} \simeq \int_{\tau}^{\infty} p_1(x)\, dx = e^{-\tau/g}$$

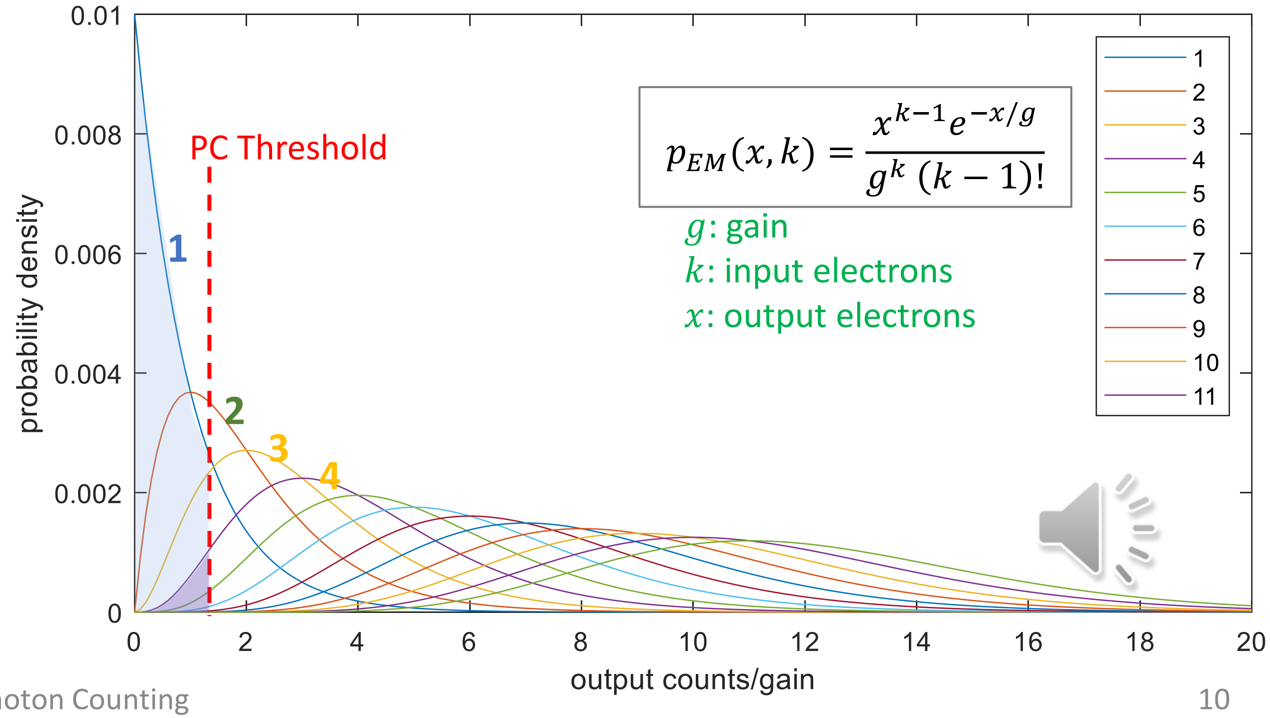
but only approximately!



Inefficiency factor ϵ_{CL}: coincidence loss

- Photon counting misses real cases where there were in fact > 1 electrons in the pixel
- This causes an undercount
- The loss is a function of the expected mean rate λ (in e-/pix/frame) in the region of interest
- This loss is accounted for, *without approximation*, by a treatment that shows:

$$\epsilon_{CL} = \frac{1 - e^{-\lambda}}{\lambda}$$



1st Order Solution to the Photon Counting Equation

- Solve the photon counting equation:

$$N_{ij} = \lambda\, n_{fr}\, \epsilon_{th}\, \epsilon_{CL}$$

$$\epsilon_{th} \simeq e^{-\tau/g}$$

$$\epsilon_{CL} = \frac{1 - e^{-\lambda}}{\lambda}$$

$$\Rightarrow$$

$$\lambda_1 = -\ln\left(1 - \frac{N_{ij}/n_{fr}}{e^{-\tau/g}}\right)$$

1st order approximation

We solve for this for each pixel; we have solved for the mean expected rate of photo-electrons. Good to ~1%!

For More On this Topic See:

PROCEEDINGS OF SPIE

SPIEDigitalLibrary.org/conference-proceedings-of-spie

Photon counting and precision photometry for the Roman Space Telescope Coronagraph

Nemati, Bijan

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SPIE.

Event: SPIE Astronomical Telescopes + Instrumentation, 2020, Online Only

Photon Counting and Precision Photometry for the Roman Space Telescope Coronagraph

Bijan Nemati[a]

[a]University of Alabama in Huntsville, 301 Sparkman Dr., Huntsville, AL 35899, U.S.A.

ABSTRACT

The Nancy Grace Roman Space Telescope will include, as one of its two instruments, the highest contrast coronagraph ever attempted, with sensitivity down to Jupiter class planets. With flux ratio below 1e-8, these planets will be exceedingl
detector. These rates nec
Roman Coronagraph will
EMCCD's, however, deliv
stochastic nature of the ele
technique called photon co
(ENF). The remaining cha
inherent to photon countin
loss, where multiple-electr
description of the photon
below 0.5%.

Keywords: photon count

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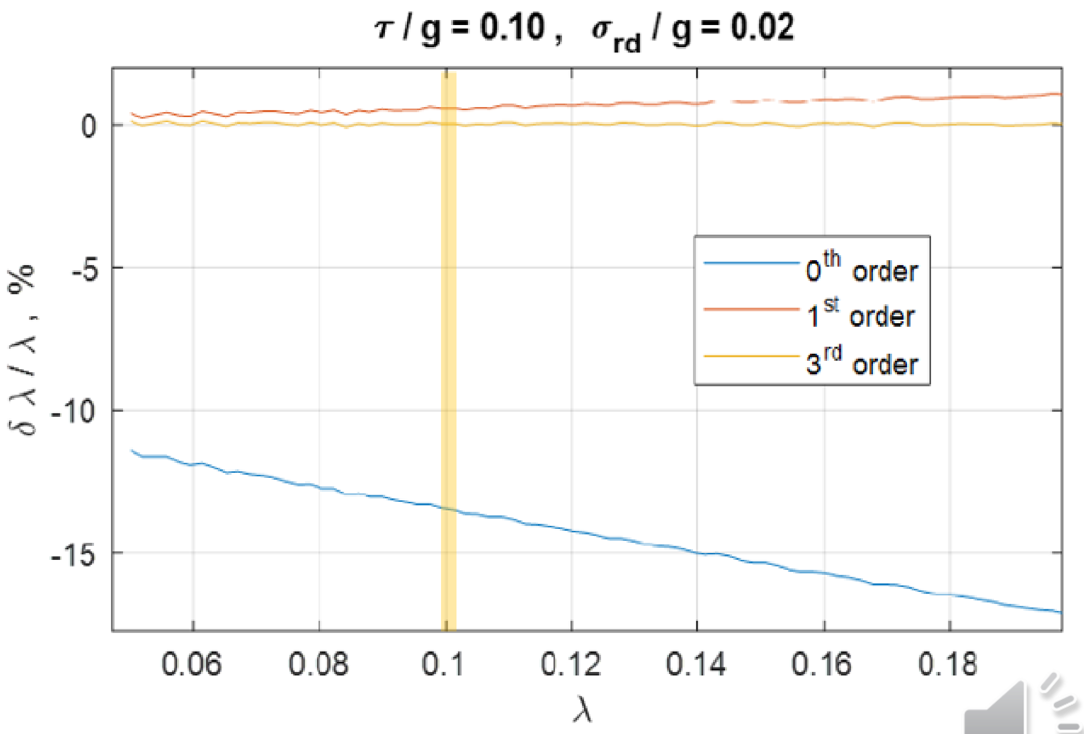
Further author informati
Bijan Nemati: bijan.nem



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CCC code: 0277-786X/20/$21 · doi: 10.1117/12.2575983

Proc. of SPIE Vol. 11443 114435F-1

Appendix

A more detailed analysis of the residual …

and the derivation of the third order correction

Examining the approximation in thresholding efficiency

$$P_p(n|\lambda) = \frac{\lambda^n \, e^{-\lambda}}{n!}$$

- There are two probability distributions at work
  - Poisson distribution associated with any given λ
  - Gamma distribution associated with any given Poisson variate
- Fraction of Poisson events *truncated* by only considering event multiplicities up to *n* is:

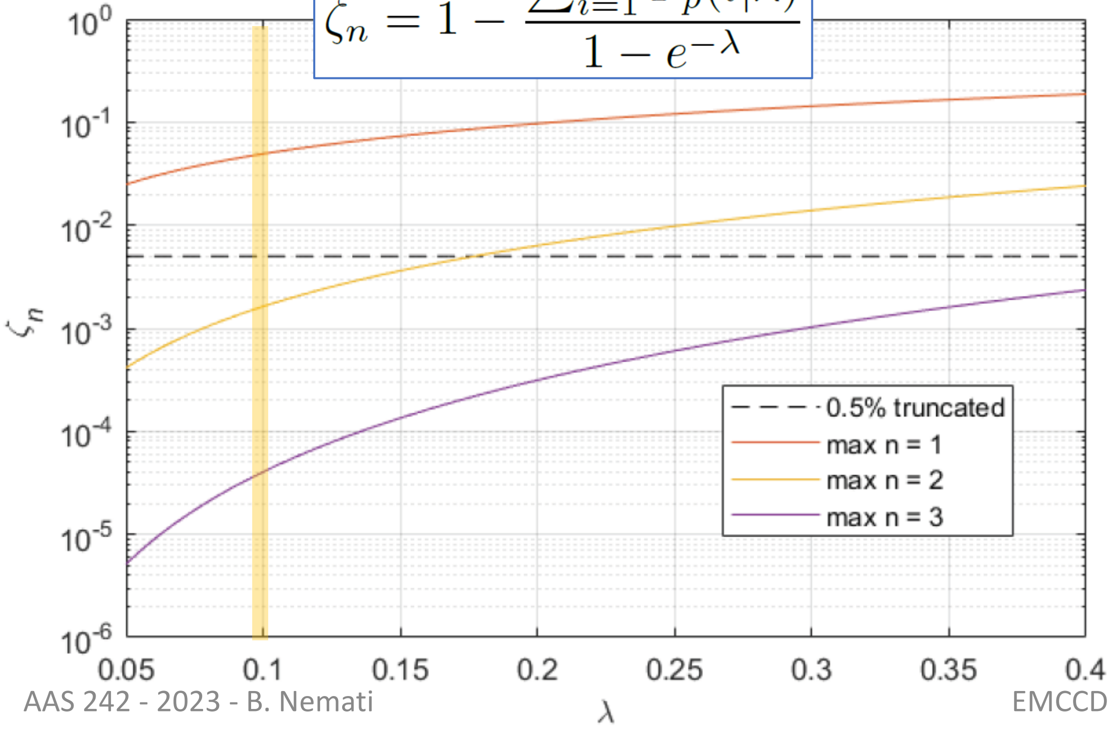
$$p(x) = \sum_n \langle x|n\rangle\langle n|\lambda\rangle$$

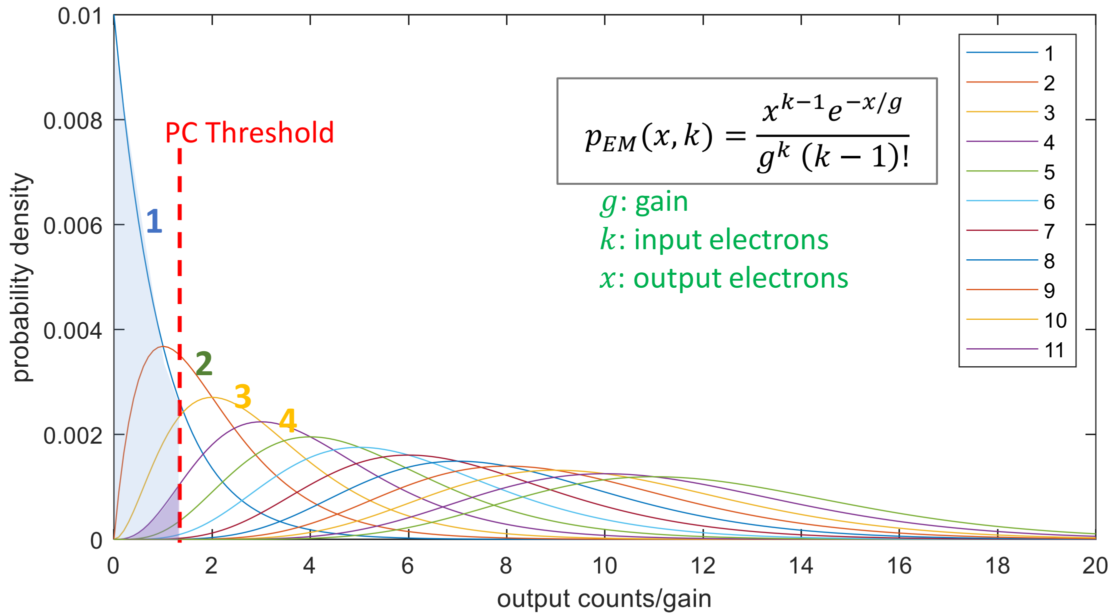
EM: $p_{EM}(x,k)$ Poisson: $p_p(k,\lambda)$

Poisson Distribution

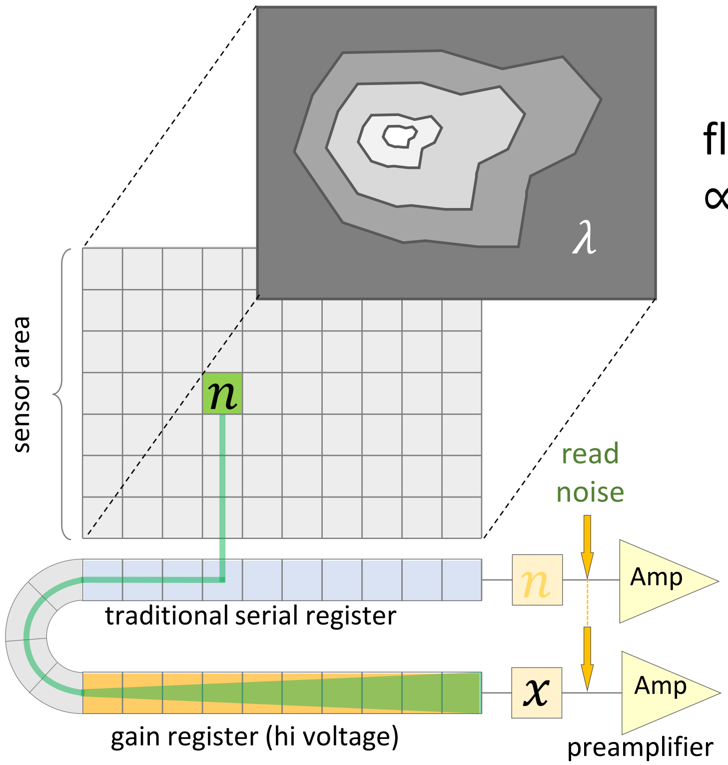
k	mean expected rate per frame, λ (c/pix/fr) 0.1	0.2	0.3	0.5	1
0	90.5%	81.9%	74.1%	60.7%	36.8%
1	9.0%	16.4%	22.2%	30.3%	36.8%
2	0.5%	1.6%	3.3%	7.6%	18.4%
3	0.02%	0.1%	0.3%	1.3%	6.1%
4	0.0004%	0.01%	0.03%	0.2%	1.5%

$$\zeta_n = 1 - \frac{\sum_{i=1}^{n} P_p(i|\lambda)}{1 - e^{-\lambda}}$$





Going from λ to x : The picture to keep in mind



flux map incident on the camera
$\propto$ a λ "brightness distribution"
λ = mean expected counts per pixel per frame

A Higher-Order Approximation of Thresholding Efficiency

- In general, the *truncated* PDF that includes terms out to a maximum *n* is given by summing over the n's:

$$P_n(x|\lambda) = \underbrace{C(n,\lambda) \cdot \sum_{i=1}^{n} P_e(x|g,i)P_p(i|\lambda)}_{P_n'(x|\lambda)}$$

$$C(n,\lambda) = \left(\int_0^\infty P_n'(x|\lambda) \; dx \right)^{-1}$$

normalize so that PDF integrates to 1

- For max n = 3, we have:
  - We can integrate to get $C(3,\lambda)$

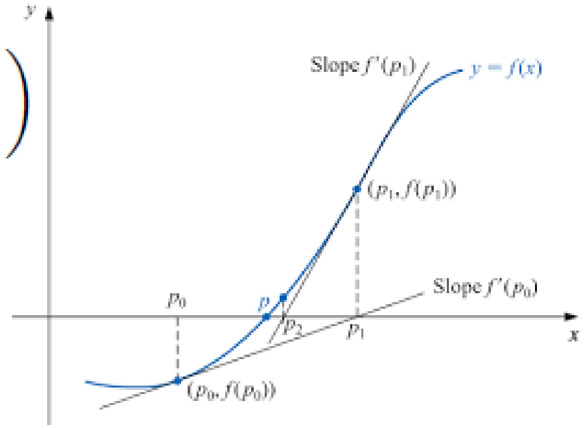
$$P_3(x|\lambda) = C(3,\lambda) \cdot \left[\lambda\, e^{-\lambda} \frac{e^{-x/g}}{g} \left(1 + \frac{\lambda x}{2g} + \frac{\lambda^2 x^2}{12 g^2} \right) \right]$$

- Then we integrate from τ to ∞ to get:

$$\epsilon_{th}^{(3)} = e^{-\tau/g} \cdot \left(1 + \frac{\tau^2\lambda^2 + 2g\tau\lambda(3+\lambda)}{2g^2(6+3\lambda+\lambda^2)} \right)$$

Solving the photon counting equation with $\epsilon_{th}^{(3)}$

- The PC equation is, as before: $N_{ij} = \lambda\, n_{fr}\, \epsilon_{th}\, \epsilon_{CL}$
- ϵ_{CL} remains the same
- But the thresholding eff is now: $$\epsilon_{th}^{(3)} = e^{-\tau/g} \cdot \left(1 + \frac{\tau^2\lambda^2 + 2g\tau\lambda(3+\lambda)}{2g^2(6+3\lambda+\lambda^2)}\right)$$
- This can be solved iteratively using the Newton method: $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



- Set an objective function whose root is the λ of interest: $$f(\lambda) = \lambda\, n_{fr}\, \epsilon_{th}^{(3)}\, \epsilon_{CL}(\lambda) - N_{ij}$$
- Need also the derivative of this:

Thank you, Mathematica !!

$$f'(\lambda) = \frac{e^{-\tau/g-\lambda}N_{fr}}{2g^2(6+3\lambda+\lambda^2)^2)} \cdot (2g^2(6+3\lambda+\lambda^2)^2 \quad + \quad t^2\lambda(-12+3\lambda+3\lambda^2+\lambda^3+3e^\lambda(4+\lambda))$$
$$+ \quad 2gt(-18+6\lambda+15\lambda^2+6\lambda^3+\lambda^4+6e^\lambda(3+2\lambda)))\,.$$

The starting guess is well supplied by our 1st order solution!

How well does this third-order solution work?

- Start with 1st order, and do 2 iterations of Newton method.
  - (Function available in Matlab!)

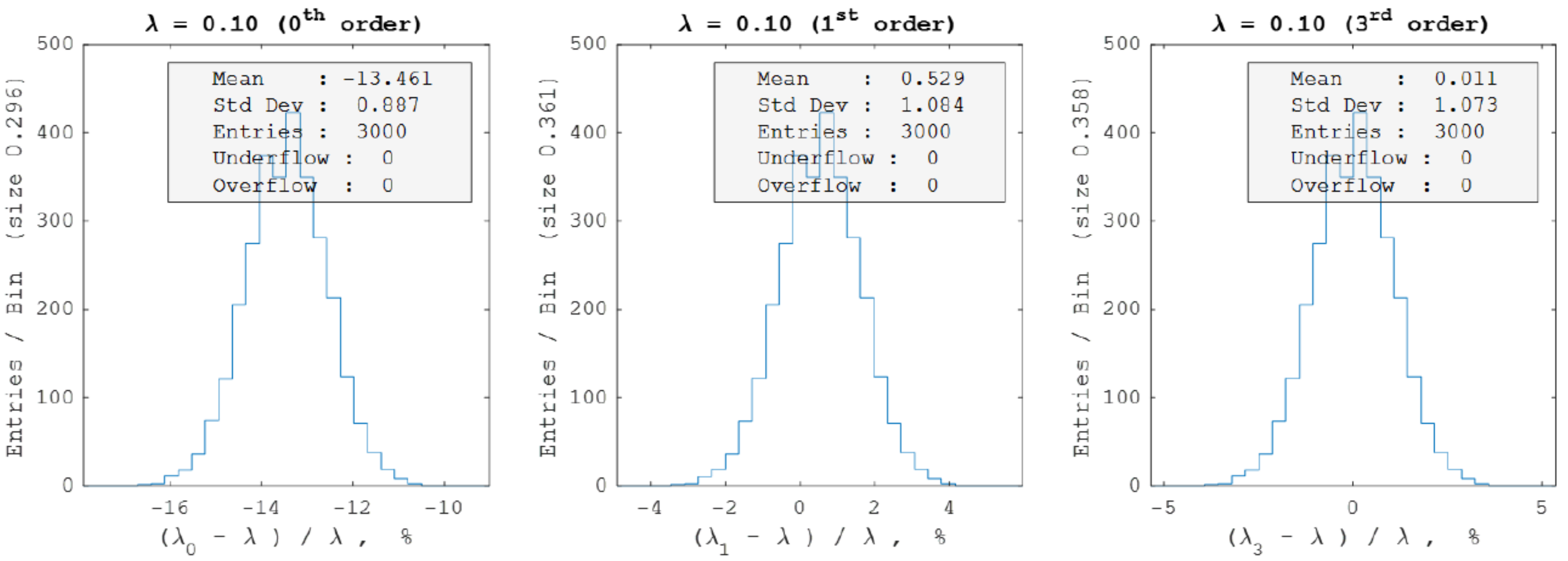


Figure 6. Testing the photometric correction algorithm on simulated pixel readouts. Each plot is a histogram of the fractional error in the estimate. On the left is the 0^{th} order estimate N_{br}/N_{fr}; in the middle is the first order estimate given by Eq. 19, and on the right is the 3^{rd} order estimate using $\epsilon_{th}^{(3)}$ and Newton's method. The means errors are seen to be 13.5%, 0.53%, and 0.01%, respectively.

Comparing the 3rd order solution with 1st order – II

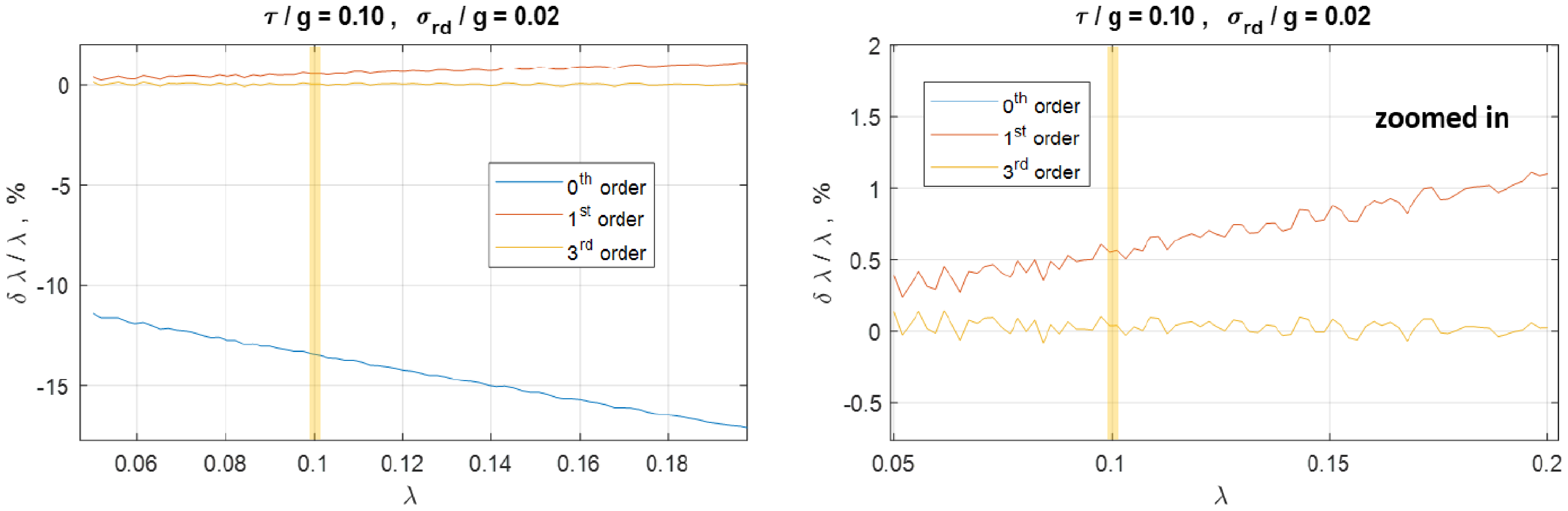


Figure 7. Testing the sensitivity of the different approximations to the actual value of λ at a given threshold. A threshold to gain ratio of 0.1 was used, while the read noise was 5 times smaller ($\sigma_{rd}/g = 0.02$), amounting to a threshold at 5 σ_{rd}. The plots are in percent fractional error, and the right plot is a zoomed-in version of the left plot, showing that the third order solution shows no visible dependence, while the first order solution does.

Comparing the 3rd order solution with 1st order – III

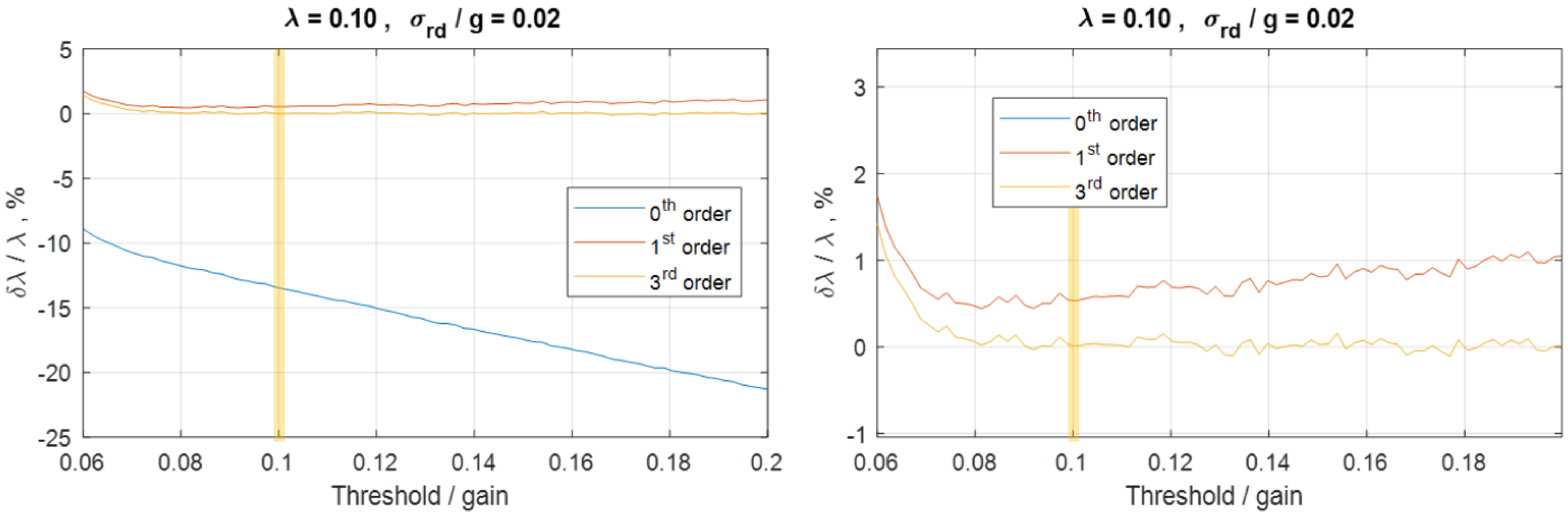


Figure 8. Threshold sensitivity of the various order approximations for estimating λ is shown, with the zoomed-in version on the right. The effect of read noise leakage is clearly seen for $\tau <\sim 4\ \sigma_{rd}$. Beyond, the third order shows no threshold dependence.