# Completeness, $\Delta$mag, and Integration Time

## Yield Modeling Tools Workshop, 2023

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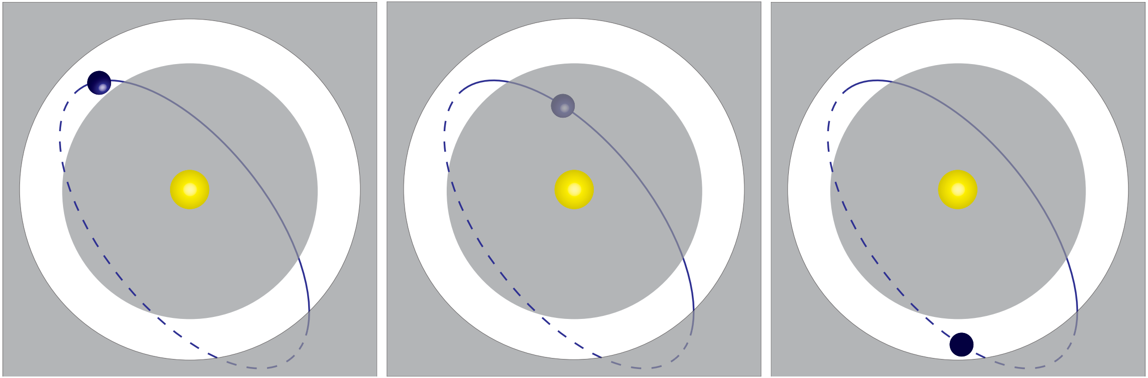
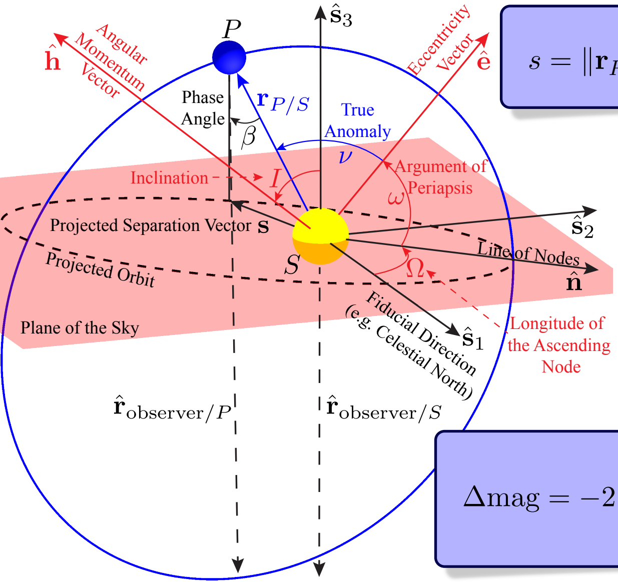
# Exoplanet Imaging in Reflected Light



Schematic of projected exosystem. Planet is sufficiently illuminated for detection in reflected light on solid part of orbit, and observable outside the gray region.

All imaging systems have an inner/outer working angle (IWA/OWA) and a limiting planet/star flux ratio (function of angular separation).

# Imaging Observables: Projected Separation and $\Delta$mag




$$s = \|\mathbf{r}_{P/S} - (\mathbf{r}_{P/S} \cdot \hat{\mathbf{s}}_3)\hat{\mathbf{s}}_3\| = r\sqrt{1 - \sin^2(I)\sin^2(\theta)}$$

Argument of latitude: $\theta \triangleq \nu + \omega$

Orbital radius: $r \triangleq \|\mathbf{r}_{P/S}\| = \dfrac{a(1-e^2)}{1+e\cos\nu}$

$a$: Semi-Major Axis, $e$: Eccentricity

$$\cos\beta \approx \frac{\mathbf{r}_{P/S} \cdot \hat{\mathbf{s}}_3}{r} = \sin(I)\sin(\theta)$$

$$\Delta\text{mag} = -2.5\log_{10}\left(\frac{F_p}{F_S}\right) = -2.5\log_{10}\left(p\Phi(\beta)\left(\frac{R}{r}\right)^2\right)$$

# The Choice of Phase Function is Important

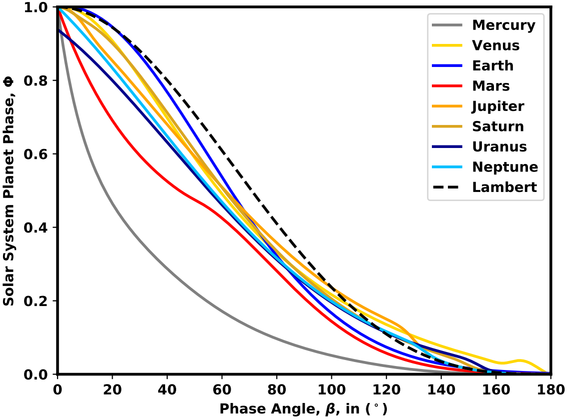



Figure from Keithly and Savransky, "The Solar System as an Exosystem: Planet Confusion", 2021 based on data from Mallama and Hilton, "Computing apparent planetary magnitudes for The Astronomical Almanac", 2018.

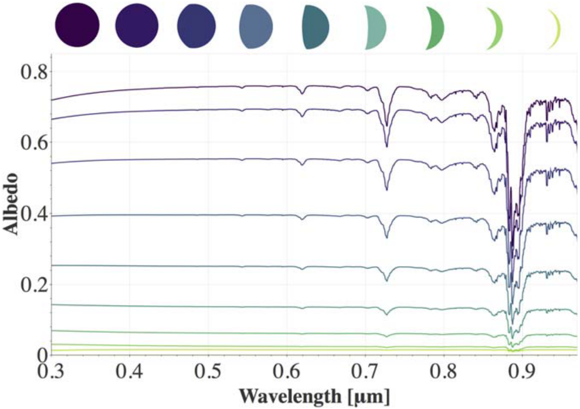
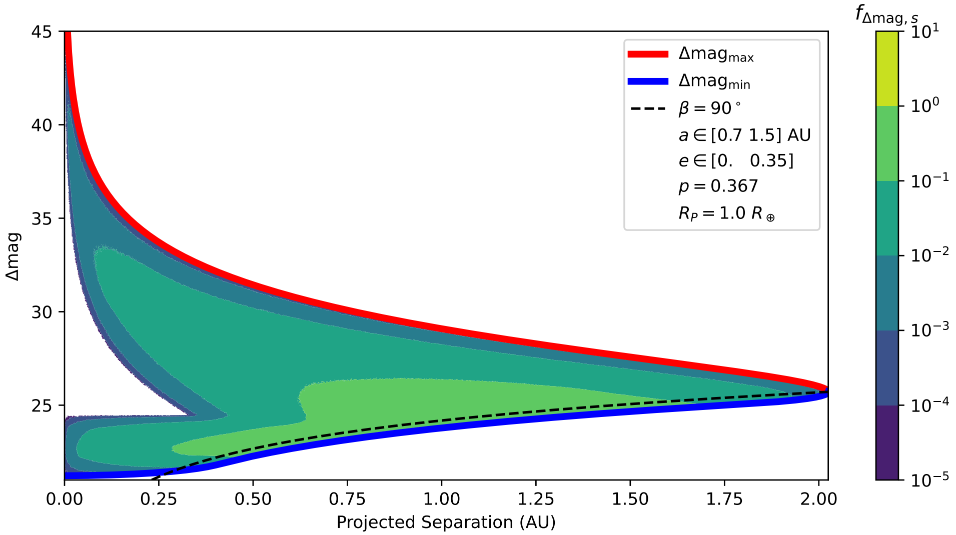


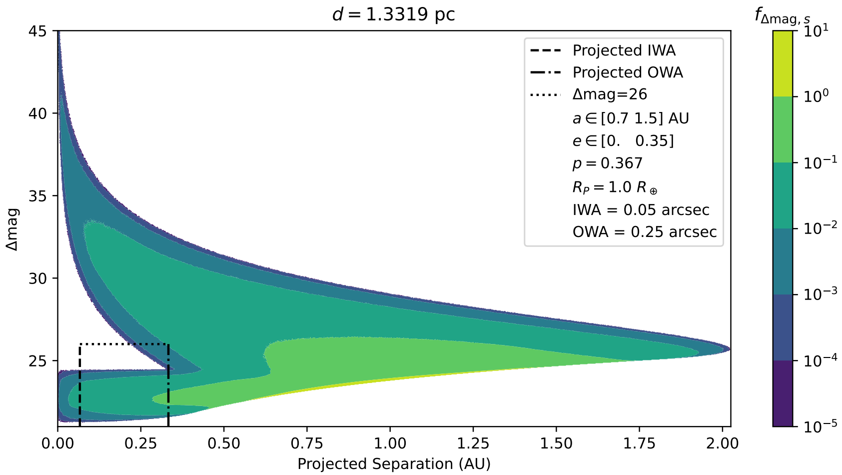

Figure from Batalha et al., "Color Classification of Extrasolar Giant Planets: Prospects and Cautions", 2018.

# The Direct Imaging Joint Probability Distribution



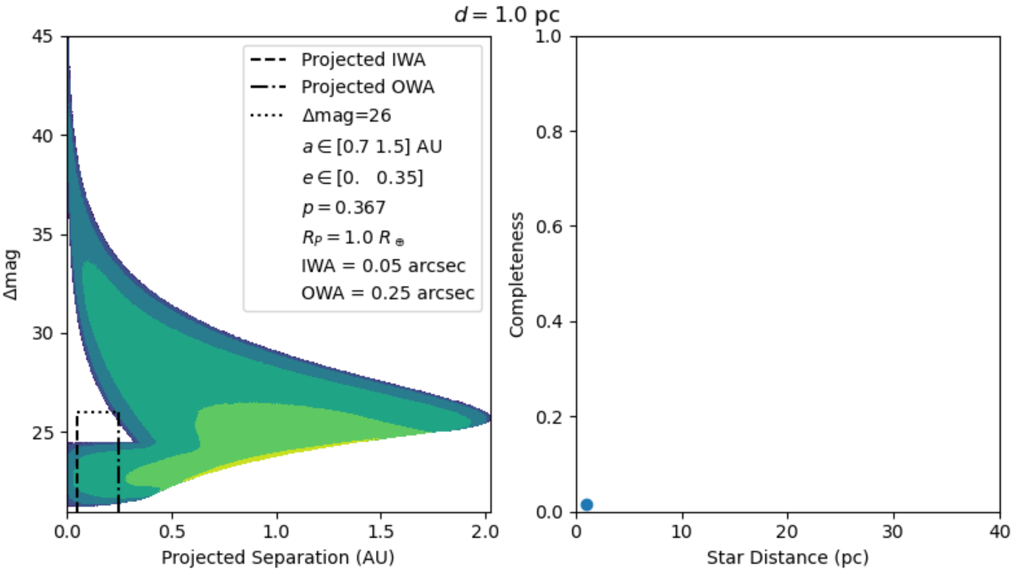

For more details see: Brown, "Single-visit photometric and obscurational completeness", 2005; Garrett and Savransky, "Analytical Formulation of the Single-visit Completeness Joint Probability Density Function", 2016

# Instrument Limits and Completeness

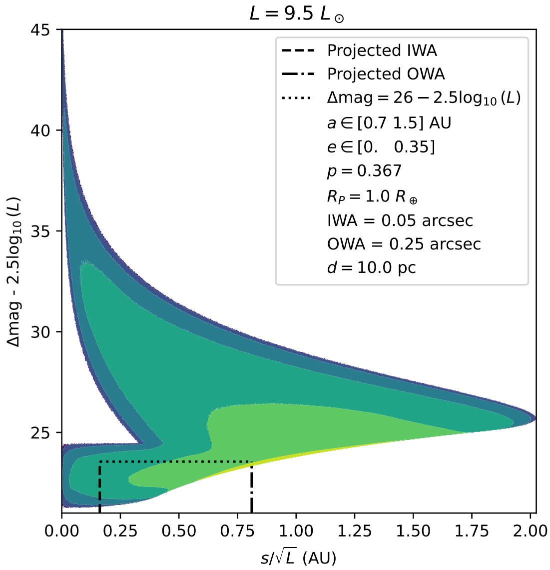



$$c = \int_{\tan(\mathrm{IWA})d}^{\tan(\mathrm{OWA})d} \int_{0}^{\Delta\mathrm{mag_{lim}}(s)} f_{\bar{s},\overline{\Delta\mathrm{mag}}}\left(s, \Delta\mathrm{mag}\right)\, \mathrm{d}\Delta\mathrm{mag}\, \mathrm{d}s$$

# The Impact of Stellar Distance



# Habitable Zone Populations and the Impact of Stellar Luminosity



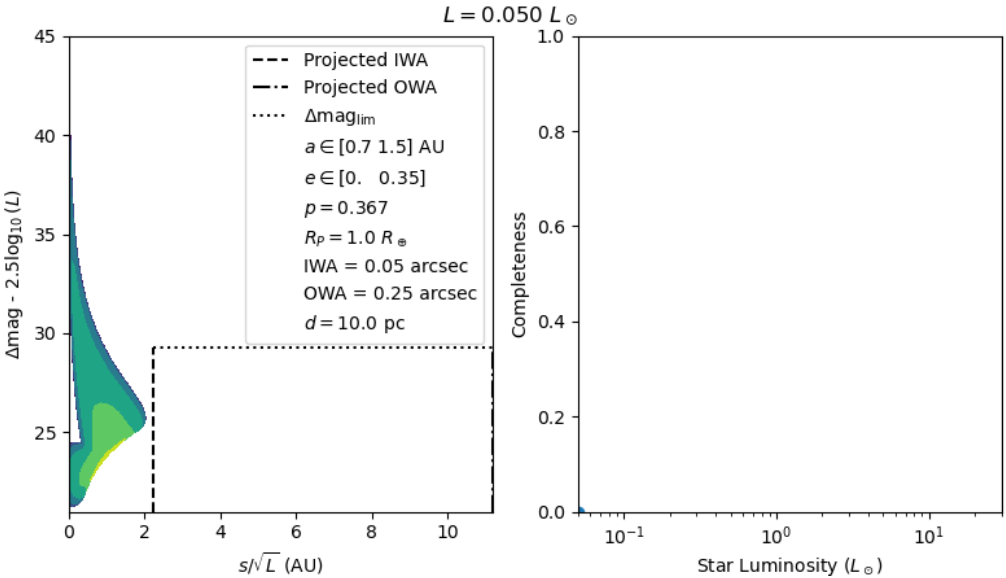

- If simulating a habitable zone population, you want to scale orbits to match solar insolation, such that $a = \sqrt{L} a_{L=1L_\odot}$ where $a_{L=1L_\odot}$ is the semi-major axis at 1 solar luminosity
- This implies:

$$r = \sqrt{L} r_{L=1L_\odot} \qquad s = \sqrt{L} s_{L=1L_\odot}$$

$$\Delta\text{mag} = \Delta\text{mag}_{L=1L_\odot} + 2.5\log_{10}(L)$$

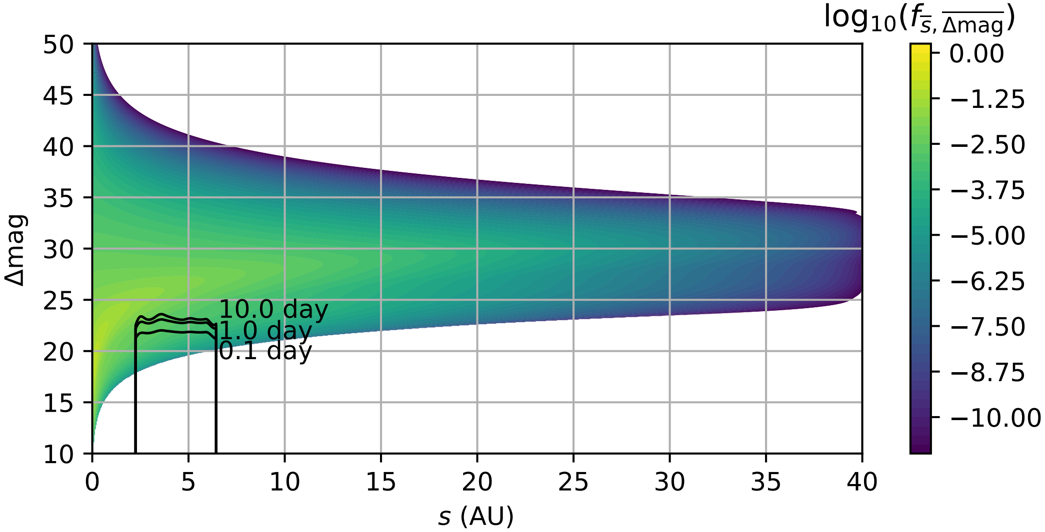
- There's no need to re-evaluate the underlying population PDF—simply redefine the abscissa and ordinate

# Habitable Zone Populations and the Impact of Stellar Luminosity

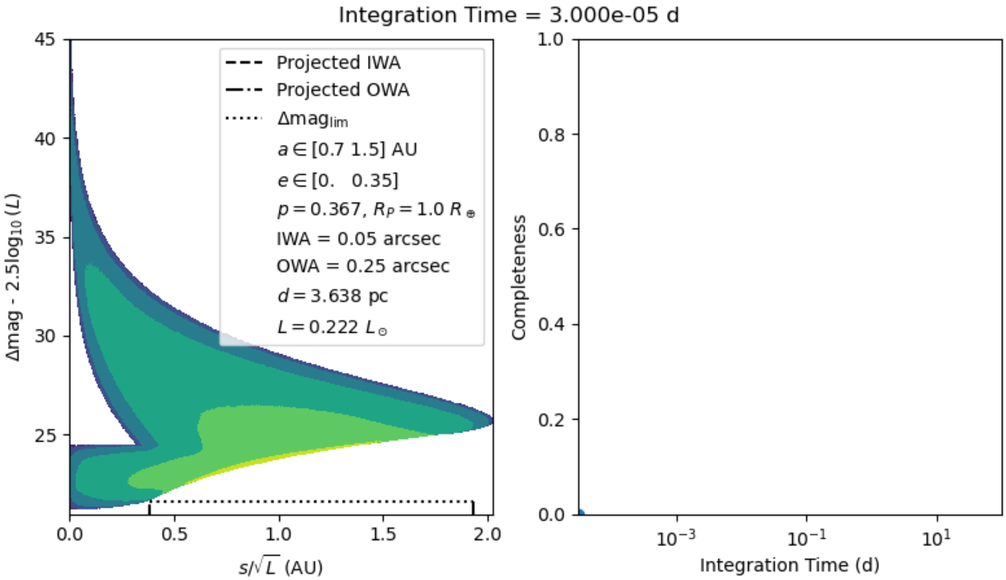


# Integration Time and Completeness

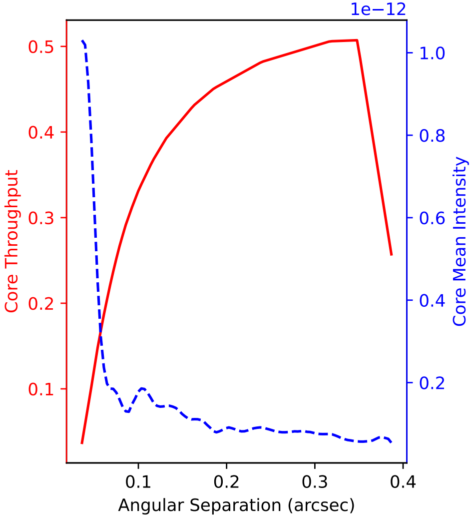
- Achievable $\Delta$mag is a function of integration time, and therefore so is completeness

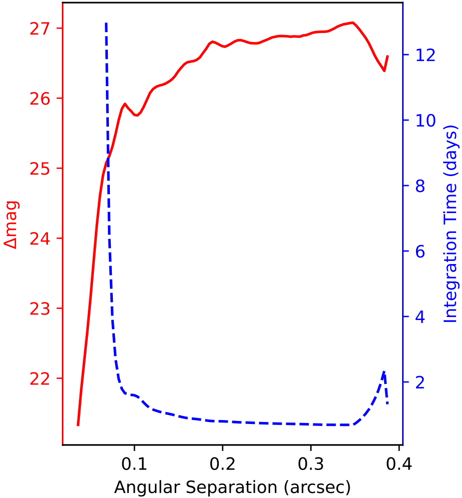


# Integration Time and Completeness

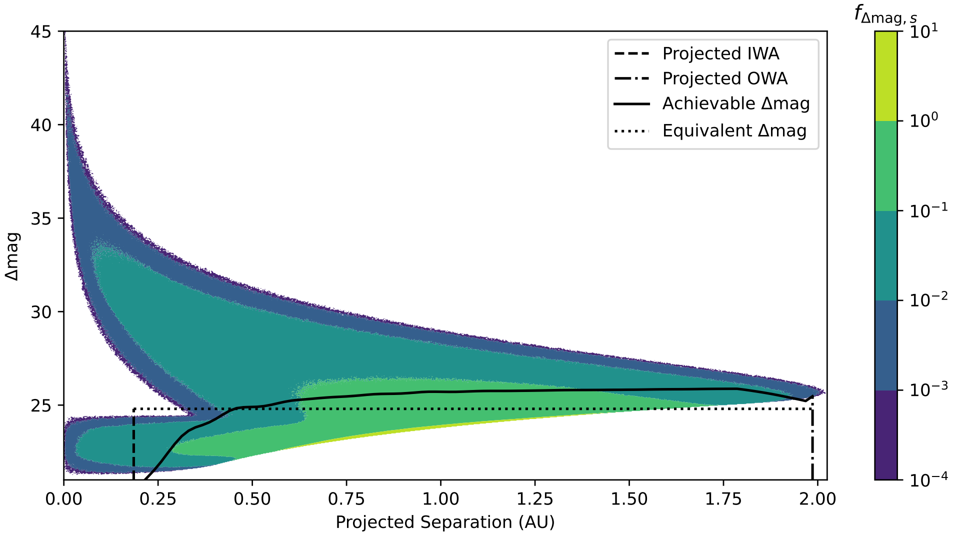


# The Impact of Real Instrument Performance

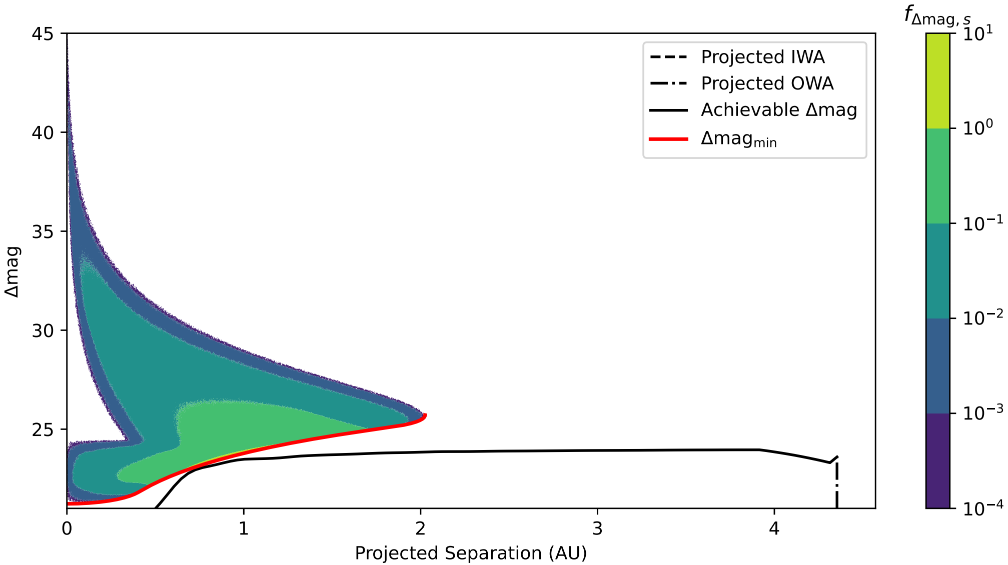


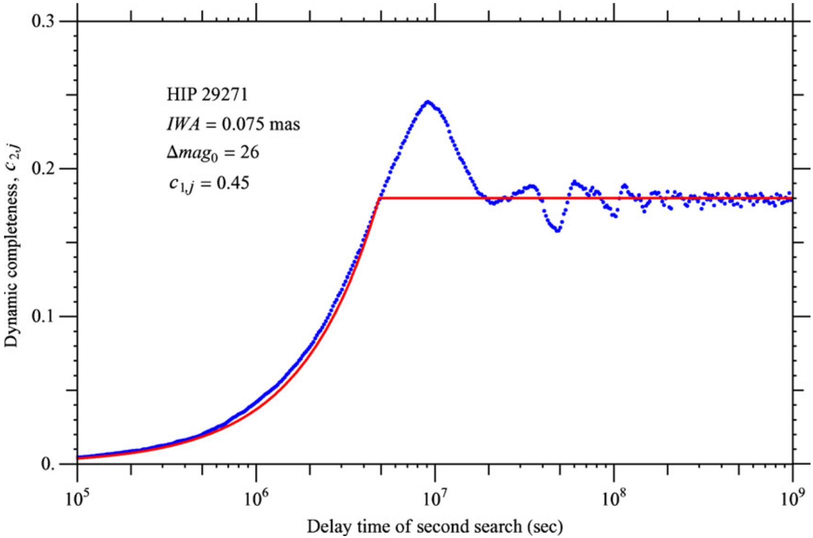
# Completeness From True Instrument Curves Can Always Be Computed with Constant Δmag limits



# Beware Edge Cases

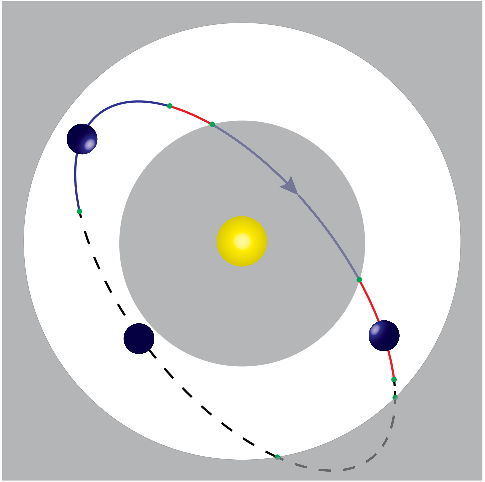


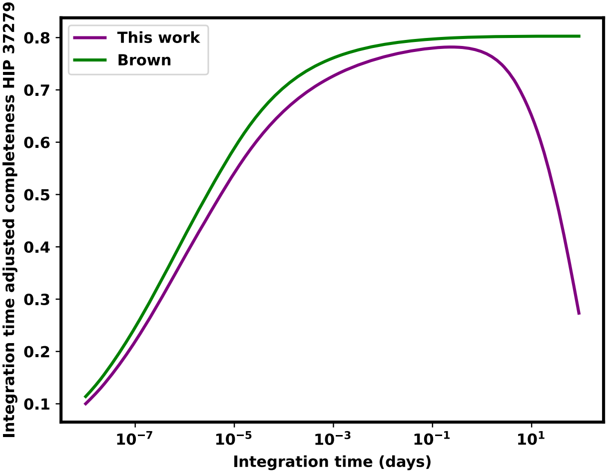
# Dynamic Completeness




From: Brown and Soummer, "New Completeness Methods for Estimating Exoplanet Discoveries by Direct Detection", 2010

# Integration Time-Adjusted Completeness






From: Keithly, Savransky, and Spohn, "Integration time adjusted completeness", 2021

# Key Take-Aways and Additional Resources

- Completeness can be computed solely by sampling semi-major axis, eccentricity, argument of periapsis, inclination, and true anomaly
- Completeness for a given population is always a function of stellar distance, but can also be a function of stellar luminosity when trying to match insolations
- Even with unlimited integration time, you may never reach a completeness of 1 due to inner/outer working angle effects and integration time saturation
- Single-visit completeness is not the end of the story: there are many refinements and extensions